

SUBSTITUTIONAL QUANTIFICATION AND ONTOLOGY

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Predicate logic deals with inferences involving *the universal quantifier*, 'All xs are F', and the *existential quantifier* 'Some xs are F', where 'F' is a predicate. The universal quantifier 'All xs' is notationally represented as $(\forall x)$, whereas the existential quantifier 'Some xs' gets represented as $(\exists x)$. In classical quantification theory (Frege-Russell-Quine tradition) these quantifiers are taken to range over domains that are classes of objects. In other words when we have a formula such as $(\exists x) Fx$, the x as a variable of this formula would get its values from a class of objects which happen to have the property 'F'. For example if 'F' stands for 'being famous', then x will pick its values from the class of famous beings; and statements like 'Fs', where s stands for Socrates, would be true instances of $(\exists x) Fx$. Quine has called this way of reading the quantifiers as objectual quantification, in order to contrast it with Lesniewski's reading of the quantifiers which takes them to be ranging over classes of 'expressions', or 'names' rather than objects.

Since objectual quantification runs into difficulties in areas such as vacuous terms and intensional contexts, logicians like Ruth Marcus, Henry Leonard, and others have invoked Lesniewski's version of quantification, which Quine nicknamed as substitutional. One of the primary motives for Marcus and Leonard was to dissociate the so-called existential quantifier from the questions of ontology. Recently, however, more than one author, have challenged

the purported ontological neutrality of substitutional quantification. In addition to that, two other major problems have emerged in recent discussions which substitutional quantification must cope with before it can have any claim to being a genuine alternative to objectual quantification: (a) the problem of indenumerably many objects in a rich universe. And, (b) whether or not all the laws of classical set theory hold under a substitutional interpretation of the quantifiers.

This paper is confined, however, to an examination of the debate among philosophers about the ontological neutrality of a substitutional approach. I intend to argue below that substitutional quantification is ontologically neutral and recent arguments against its neutrality are basically defective.

I

Quine, who is no advocate of substitutional quantification, underscores its ontological neutrality in the following words:

.....substitutional quantification makes good sense, explicable in terms of truth and substitution, no matter what substitution class we take -even that whose sole member is the left-hand parenthesis. To conclude that entities are being assumed that trivially, and that far out, is simply to drop ontological questions. Nor can we introduce any control by saying that only substitutional quantification in the substitution class of singular terms is to count as a version of existence....[Because] the very notion of singular terms appeals implicitly to classical or objectual quantification¹.

There are two Quinean arguments for the ontological neutrality of substitutional quantification in this passage. First, if it is possible for a trivial substitution class to produce true instances (an instance results by replacing 'x' in such formulae as $(\exists x) 'Fx'$ by members of the substitution class which are expressions and are substituends for the variables.), and there is no reason why it should not be possible, then we have no criterion whatsoever for distinguishing trivial from non-trivial substitution classes. More specifically we cannot distinguish names from other vocabulary. Second, Quine points out that we cannot fall back upon the substitution class of singular terms either. The reason is that singular terms refer to objects and hence lend themselves more naturally to objectual quantification.

Parsons [1971] and Orenstein [1984], however, for varying reasons do not accept the claim that substitutional quantification is ontologically neutral. Parsons believes that "the existential quantifier substitutionally interpreted has a genuine claim to express a concept of existence which has its own interest and which may offer the best explication of the sense in which "linguistic" abstract entities - propositions, attributes, classes in the sense of extensions of predicates - may be said to exist."² In order to substantiate his claim Parsons makes an effort to answer Quine's arguments.

With respect to the first argument Parsons points out two formal features of the category of singular terms which clearly distinguish it from such trivial substitution classes as the one whose sole member is a left-hand parenthesis. First, the category of singular terms "admits identity with the property of substitutivity *salva veritate*. Second, it has infinitely many members that are distinguishable by the identity relation."³ This, of course, won't do

unless Parsons takes care of Quine's second argument that "the very notion of singular terms appeals implicitly to classical or objectual quantification". But before we go into that it seems worthwhile to examine a recent criticism of Parsons' response to Quine's first argument.

Gottlieb and McCarthy [1979] find Parsons' appeal to the two formal features of the category of singular terms unconvincing. They declare that "the role of identity in a substitutionally interpreted language is quite different from its ordinary role. As Ruth Marcus pointed out, identity will be replaced by a series of syntactically defined substitution principles depending for their scope on the expressive richness of the language."⁴

Gottlieb and McCarthy do not specify the nature of these substitution principles and, it seems to me, that they are involved in a slight misrepresentation of Marcus' position; a misrepresentation which is of some consequence for our present discussion.

One of the ways by which Ruth Marcus draws a distinction between intensional and extensional languages is that in the latter we equate identity with some weaker form of equivalence relation. Thus, for example, in systems where identity is not taken as primitive, we define it in terms of indiscernibility.

$$x = y \text{ Df } (\varphi) (\varphi x \equiv \varphi y)$$

This, for Ruth Marcus, is an extensionalizing principle because we are equating identity with a slightly weaker relation of indiscernibility. But at the level of individuals this is the only extensionality principle.

We cannot equate identity at this level with any equivalence relation other than indiscernibility.⁵ In other words Marcus distinguishes identities like:

(a) 'The number of planets = 9', from "true" identities wherein identity sign flanks two names. The reason is that 'the number of planets' is not substitutable for '9' in intensional contexts, such as

(b) ' $\square 9 > 7$ '. Names, however, are intersubstitutable in all contexts except where we are mentioning a name instead of using it.⁶

Furthermore, as Marcus points out, it is possible for singular descriptions to acquire the status of proper names. "Singular descriptions such as 'the little corporal', 'the Prince of Denmark', 'the sage of Concord', or 'the great dissenter', are as we know often used as alternative proper names of Napoleon, Hamlet, Thoreau and Oliver Wendell Holmes."⁷ Hence wherever a singular description behaves like a name the ordinary role of identity in regard to it won't undergo any change.

Clearly then, Gottlieb and McCarthy are wrong in suggesting that the role of identity in substitutional languages is different from its ordinary role with regard to all singular terms. If Parsons can so qualify his substitution class as to be comprised of names alone (and that is all he needs for substitutional quantification) then the two formal features of singular terms that he speaks of would enable him to distinguish his substitution class from any trivial ones. And this blocks Quine's objection successfully. We must turn now to Parson's response to Quine's second argument that the very notion of singular

terms appeals implicitly to objectual quantification.

Parsons willingly concedes Quine his basic point here. He says:

We can concede Quine's point here for a certain central core class of singular terms, which we might suppose to denote objects whose existence we do not expect to explicate by substitutional quantification. We might then make certain analogical extensions of the class of singular terms in such a way that they are related to quantifications construed as substitutional. The criterion for "genuine reference" is given in other terms.⁸

Parsons explains his "analogical extensions" by pointing out "a natural way to introduce a predicative theory of classes (extensions of predicates)." If 'F' stands for a one-place predicate of some first-order language we can rewrite 'Ft' as ' $t \in \{x:Fx\}$ '. Now if we take ' α ', ' β ' as schematic for class abstracts such as ' $\{x:Fx\}$ ' we can define ' $\alpha = \beta$ ' as ' $(\forall x) (x \in \alpha \equiv x \in \beta)$ '. We can then allow the abstracts to be replaced by quantifiable variables of a new sort. And substitutional interpretation would provide us with truth conditions for the resulting formulae.⁹ In other words, analogical extensions such as class abstracts would comprise our substitution class now. And as these abstracts do not denote any objects, they lack any appeal to objectual quantification.

Parsons claims that given suitable intensional equivalence relations this procedure would go through for attributes or any other terms. His conclusion is that "in the case where the terms involved

have a non-trivial equivalence relation with infinitely many classes, substitutional quantification gives rise to a "genuine doctrine of being" to be set alongside Quine's and others".¹⁰ Parsons does not state explicitly as to how this "doctrine of being" rises. But let us see what his critics say.

Gottlieb and McCarthy, once again, are dissatisfied with Parsons' response to Quine. They point out that the reason Quine employs an objectual ' $(\exists x)$ ' as a touchstone for a genuinely referential use of proper names and definite descriptions lies precisely in the fact that "there is no purely syntactical criterion for terms which are used to denote objects". Parsons' "analogical extensions" are concerned with syntax - giving us reasons to classify class abstracts etc. with singular terms - and do not seem to establish any reference. It is obvious that

(x) (the museum has a statue of x)

would be true on a substitutional interpretation even if the only statue in the museum is that of Zeus. If this does not warrant us to accord any existence to Zeus then the question Gottlieb and McCarthy ask is: why should "the truth of

$$(\exists a) (\forall x) (x \in a \equiv x \text{ is a horse})$$

lead us to accord any sense of existence to

$$\{x : x \text{ is a horse}\}.$$
¹¹

Gottlieb and McCarthy believe that perhaps Parsons relies on the presumption that quantification at the metalinguistic level is objectual and therefore our interpretation of quantified sentences commits us ontologically to expression-types. They claim that quantification at the metalinguistic level need not be objectual, and

furthermore, there is no reason for us to associate with a sentence S the commitments of S^* which gives truth conditions for S.¹²

Now it seems to me that Parsons' paper nowhere suggests a purely syntactical criterion of ontological commitment. Indeed such a possibility is entirely ruled out if he presumes what Gottlieb and McCarthy suggest that he does, namely, that the quantification at the metalinguistic level is objectual. Parsons is aware of the difficulty involved in such a recourse and says that if we resort to objectual quantification at the metalinguistic level we are no longer doing purely substitutional quantification. But if we remain substitutional at every level we get involved in a regress that we must end somewhere.

The fact of the matter, however, is that without going objectual at some metalinguistic level substitutional quantification does not give rise to Parsons' purported "doctrine of being". The reason for this is quite simple. Substitutional quantification makes sense only if our substitution class is non-empty. For otherwise, as Linsky remarks (Orenstein [1984] makes the same point) "all universal quantification would be false merely for the lack of any substitution instances of open sentences at all".¹³ This means that in order for us to have an acceptable semantics at all we must have expressions in our substitution class. (And if it is Parsons' "abstracts" that comprise our substitution class so be it). However, we need not resort to objectual quantification at metalinguistic level. We can have another substitution class comprised of suitable expressions. And this is true about every level. Now while it is true that we cannot have substitutional quantification without a non-empty substitution class, there is nothing to compel us to resort to objectual quantification at any level. If so, then we need not be

taken to be committed to anything other than purely syntactical classes. This, of course, is a far cry from a genuine 'doctrine of being' that Persons' talks about. The central point that I want to make is: why can't we remain substitutional at every level, and why can't we live with the resulting infinite regress? After all even in case of objectual quantification such a regress is present because truth of statements of a given level is always stated in the next higher level and these levels can continue all the way to infinity.

II

Orenstein (1984) is the second author I wish to briefly consider here on the question of ontology and substitutional quantification. Apart from reiterating Linsky's point hinted at earlier the main point of his argument revolves around a distinction between truth conditions of substitutional quantification on the one hand and truth conditions of instances involved therein on the other. He insists, and I think rightly, that we must answer the question as to how our instances acquire their truth. We cannot take the truth of instances such as "Alfred is human" for granted because that "resembles truth by convention". The only two other possibilities, according to Orenstein, are referential semantics and the model theoretic approach. He dismisses the model theoretic approach simply by saying that "to make consistency the nature of truth smacks of the coherence theory of truth."¹⁴ The only reasonable choice for us, according to Orenstein, then is to interpret our instances referentially.

Now a quantifier is referential for Orenstein "either because the interpretation of the quantifier is referential (Tarski) or the instances on which the generalization is based are interpreted referentially (Leblanc)". In other words if we interpret atomic

sentences (instances) referentially our quantification becomes "substitutional and referential". Obviously such an approach is not neutral ontologically.

Where neither the quantifier nor the instances are interpreted referentially, and a model theoretic approach is adopted to explain the truth of instances, Orenstein believes, we can avoid ontological commitment. "The key point is that such an instance without referential force bestows no referential-ontological significance upon a substitutional generalization based on that instance. So where the instances are not construed referentially, the substitutional quantification has no such force."¹⁵ The only trouble that Orenstein sees with a model theoretic approach is that it makes consistency the nature of truth which smacks of the coherence theory. Orenstein calls such a quantification as "non-referential and substitutional".

I have only two observations to make regarding Orenstein's position. The first pertains specifically to Ruth Marcus' model theoretic approach. I do not think that the model proposed by her avoids word-object relation (reference) altogether, although it places it at a different level and succeeds in maintaining the ontological neutrality of substitutional quantification. Let us look into her model. She defines model of language L as a class of ordered pairs of D where D is the domain of individuals named by the constants of L. (In addition to individual constants L has an infinite number of individual variables, truth functional connectives, one two-place predicate R, quantification, and criteria for being a wff.) The membership of model (M) would be exactly those pairs between which R holds. A sentence $R(a_1, a_2)$ would be true in M iff the ordered pairs (b_1, b_2) is a member of M, where a_1 and a_2 are names in L of b_1 and b_2 . If a sentence A has the form $\sim B$ in L it will come out to be true in M if B is false in M. Likewise, if A has the form $(\exists x) B$ then

A comes out to be true in M if at least one substitution instance of B is true in M.¹⁶

At the face of it in this model the truth of an instance gets defined in terms of model membership (consistency with other sentences of the model) with no direct recourse to things. But membership of pairs in the model is contingent upon R holding between individuals that comprise pairs. And there is no way for us to determine whether or not R holds between any two individuals without a recourse to word-object relation. This shows that Ruth Marcus' model theoretic approach for the interpretation of instances cannot completely avoid word-object relation. It simply places it at a different level. Still, however, it must be granted that the word-object relation or reference does not arise in Marcus' approach because of substitutional quantification, which her model-theoretic approach succeeds in keeping ontologically neutral.

My second observation regarding Orenstein's position pertains to his distinction between truth conditions of substitutional quantification on the one hand, and truth conditions of instances of quantified statements on the other. As I hinted above Orenstein is right when he insists that we cannot take the truth of instances of formulae like ' $(\exists x) Fx$ ' as for granted. We cannot. However, we must insist on a Tarskian definition of referential quantifier, i.e., it is referential when interpreted referentially. Orenstein expands the definition of referential quantifier on the authority of Leblanc by saying that if we interpret atomic sentences (instances) on which a generalization is based as referentially, our quantifier becomes referential. This expansion of the notion of referential quantifier seems suspicious. Whether or not the word-object relation or reference obtains at the level of instances of a quantified statement

has got nothing to do with the referentiality of the quantifier itself. A quantifier would be referential only if the domain it ranges over is comprised of objects. In other words when its values happen to be objects. If that is not the case and its values are 'expressions', then the truth of the resulting instances like 'Alfred is human' need not affect the ontological neutrality of the quantifier. The word-object relation that might be involved in the truth of such sentences as 'Alfred is human' is at one remove from the ontological commitment or otherwise of the quantifier. Of course, we need not be conventionalists about the truth of such sentences but that's a different matter altogether.

I conclude, therefore, that substitutional quantification does succeed in achieving ontological neutrality.

NOTES

1. W.V. Quine, *Ontological Relativity and other essays*, 1969, P.106.
2. Charles Parsons, A Plea for Substitutional Quantification, *Journal of Philosophy*, (68) 1971, PP. 231-7.
3. Ibid.
4. Dale Gottlieb and Timothy McCarthy, Substitutional Quantification and Set Theory, *Journal of Philosophical Logic*, (8) 1979, PP.315-331. Also see Dale Gottlieb, *Ontological Economy: Substitutional Quantification and Mathematics*, 1980, PP. 53-7.
5. Ruth Marcus, Extensionality, in *Reference and Modality*, ed. L. Linsky, 1971, PP. 44-51

6. L. Linsky, OP. Cit., P.3.
7. Ruth Marcus, Modalities & Intensional languages, in *Studies in the Philosophy of Science*, ed. M. Wartofsky, 1963, PP. 77-96.
8. Charles Parsons, OP. cit.
9. Ibid.
10. Ibid.
11. Dale Gottlieb and Timothy McCarthy, OP. cit.
12. Ibid.
13. L. Linsky, OP. cit. P. 97.
14. Alex Orenstein, Referential and Nonreferential Substitutional Quantification, *Synthese*, (6) 1984, PP.145-157.
See also his book *Existence and the Particular Quantifier*, 1978, P. 37 for the claim that there is no reason to adopt objectual quantification at metalinguistic level in substitutional quantification. Dunn and Belnap, The Substitution Interpretation of the Quantifiers, *Noûs*, 1968, PP. 177-185, are the first to make this claim.
15. Op. cit., 1968.
16. Ruth Marcus, Op. cit., 1963.